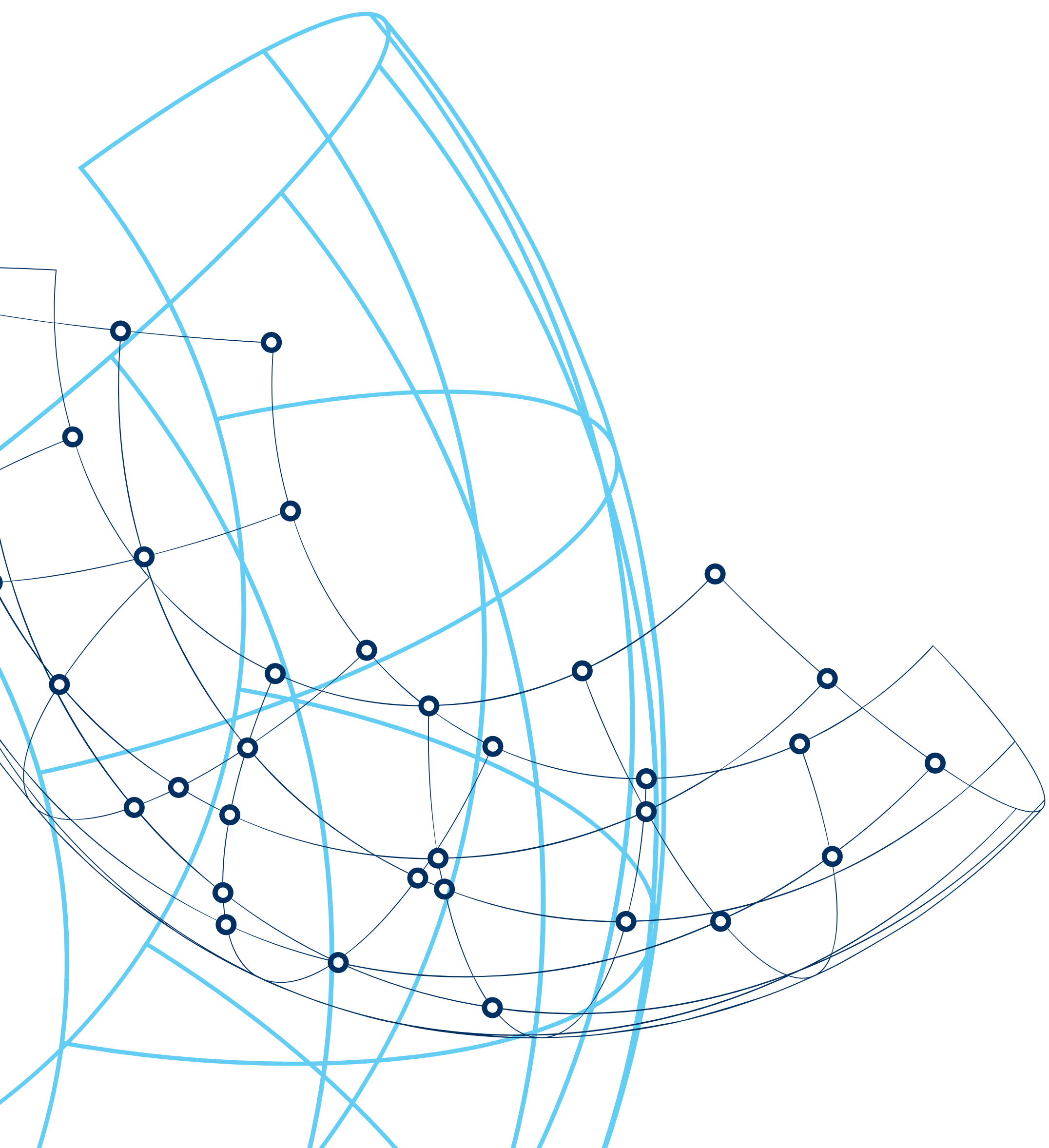
Swap netting using a quantum annealer



White Paper

Swap netting using a quantum annealer

Gili Rosenberg, Clemens Adolphs, Andrew Milne, and Andrew Lee

Abstract

Swap trades that are cleared through a clearing house may be netted against each other. By doing this, the clearing house reduces its risk exposure, and the counterparties regain the use of capital that was previously tied up in margin accounts. The simplest form of netting is to cancel trades that offset each other exactly. However, it is also possible to net trades, or chains of trades, that sum to a very small residual. The ability to find new nettable combinations can lead to new capital efficiencies. The 1QBit swap netting solution makes use of a quantum annealer to identify such combinations and presents them as a netting proposal. The candidate swaps are chosen based on an incompatibility function, which incorporates differences in economic terms in a flexible way.

Keywords: swap netting, quantum annealing, optimization

1 Problem description

A swap is an agreement to exchange cash flows at specific dates, for a specific term. When a swap is cleared, the original agreement between (say) counterparty A and counterparty B is converted (novated) by the clearing house into two separate agreements: one between the clearing house and counterparty A, and another between the clearing house and counterparty B. The clearing house protects each counterparty from the risk of default by the other. It holds collateral from both counterparties, in an amount determined in part by the gross notional value of their cleared trades. A clearing house may have to guarantee a large number of cash flows for a very long time (up to thirty years), creating the situation illustrated (in simplified form) in Figure 1.

In this white paper, we solve a problem in which all of the swaps are plain vanilla interest rate swaps defined by a fixed-rate leg and a floating-rate leg. However, our method can be generalized to other types of tradable instruments for which netting can be defined. We solve the problem from the standpoint of the clearing house, such that the swaps for each counterparty can be analyzed and processed independently. The problem of netting agreements between multiple counterparties, for example by a third-party netting service, is left for future work.

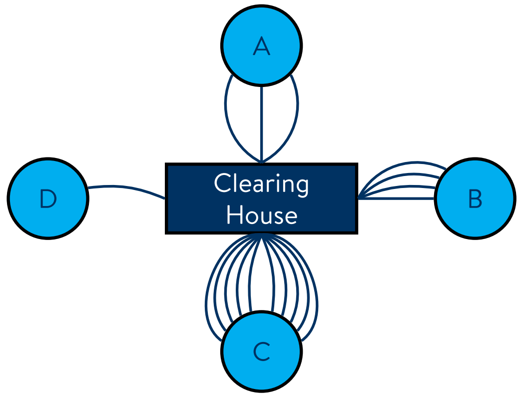


Figure 1: A clearing house that has multiple swaps with multiple counterparties.

2 Our solution

We begin by considering each counterparty independently, as illustrated in Figure 2.

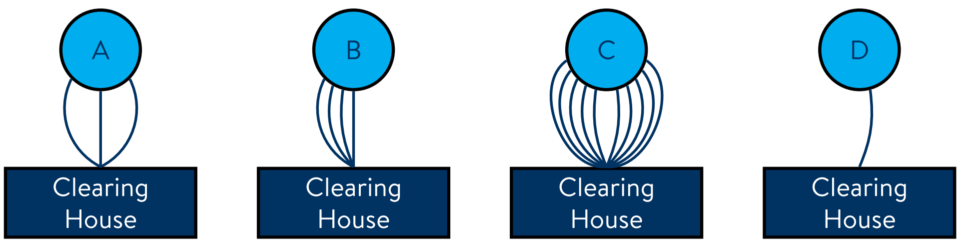


Figure 2: Each of the clearing house's counterparties' swaps can be netted separately.

We consider a set S of swaps between the clearing house and a given counterparty. Our netting solution consists of three basic steps:

1. Partitioning S into subsets, each of which contains swaps that can be netted against each other.
2. Calculating a netting proposal for each subset.
3. Aggregating the subset netting proposals into a netting proposal for the full set S.

Historically, swap netting began with swaps that had identical economic terms, which could be thought of as a very fine partition of S. Later, it became possible to combine swaps with different fixed rates in a process known as coupon blending. In general terms, as the calculation of netting proposals becomes more sophisticated, the partitioning of S can be broadened. Our netting solution anticipates an extension of this process into larger-scale partitions where the number of potential netting combinations in each subset of the partition grows exponentially. In the present white paper, we describe a separable solution, in which the set S is partitioned into subsets, and netting proposals are calculated for each subset separately. However, our solution could be modified such that it optimizes simultaneously over both the choice of partition and the aggregated netting proposals, in principle leading to better netting proposals.

Consider again the set S of swaps for a given counterparty. We introduce the partition P on S, where $S = \bigcup_{M_k \in P} M_k$, such that M_k is a subset of S. For a given M_k, the elements will be swaps that are similar enough to be netted with each other. In a fine-grained partition these may be swaps with identical economic terms. However, our intent is to consider partitioning into a smaller number of larger subsets, and to use the quantum annealer to select nettable groups within them. For convenience, we drop the index k for the remainder of this section, and describe the selection process within a single subset M. The objective is to select subsets of M with a maximum notional that can be netted, such that within each subset, the fixed legs and float legs are similar to each other. We denote the notional of swap i by N_i, and define the direction by d_i, such that it is $+1$ if the clearing house pays the fixed cash flow to the counterparty and -1 if the counterparty pays the clearing house the fixed cash flow.

In order to solve this problem using a quantum annealer, we need to formulate it as a quadratic unconstrained binary optimization (QUBO) problem. We define a binary decision variable x_i for each swap i in the initial pool of swaps M, before any swaps are netted. The binary variable's value is one if the corresponding swap is in the chosen group of swaps to be netted and zero otherwise. The optimization problem for choosing the swaps to be netted can be written as

$$x = \text{argmax}_x \left[\sum_{i \in M} x_i N_i - A \left(\sum_{i \in M} x_i d_i N_i \right)^2 - B \sum_{i,j \in M} x_i x_j I_{ij} \right]. \tag{1}$$

The first term maximizes the total notional of the swaps in the group, the second term penalizes swap groups with notionals that are far from cancelling out, and the third term penalizes pairs of swaps based on their pair-wise incompatibility. The non-negative incompatibility I_{ij} must be calculated in advance for every two swaps. One could view the incompatibility function as prioritization: weights could be given to the difference of each term (such as interest rate, duration, etc.) based on prioritization by the clearing house. The constants A and B set the relative importance of each of the terms and can either be provided manually based on theoretical considerations or computed, for example via the sub-gradient descent method. The process described above for the generic part M is repeated for all of the M_k's that make up the partition P.

It should be noted that groups of similar but non-identical swaps may not be completely nettable. In coupon blending, for example, the calculation used to balance the cash flows can lead to interest rates that are away from current market values. This raises the point that a different partitioning could lead to different groups being found, and the question of whether searching over a set of partitions could lead to a globally optimum netting proposal for S as a whole. For a particular subset, the quantum annealer returns a sample of solutions, which could potentially lead to multiple different netting proposals for that subset. In addition, we could get multiple netting proposals by varying the constants A and B. In the above coupon blending example, given multiple netting proposals, it would be possible to choose the netting proposal that yields an interest rate that is closest to current market values.

3 The 1QBit prototype

We have implemented a proof-of-concept prototype, which takes a database of existing swaps, identifies swaps to be netted using a quantum annealer, and generates a netting proposal. The entire process is logged, including a report showing the reduction in notional for each counterparty and for the clearing house. The algorithm for the prototype is presented in Algorithm 1.

Algorithm 1 Swap netting algorithm

Require: Set of swaps to be netted S
  Validate and standardize S (swap trades may be reported in more than one way)
  Apply the partition P to S (based on term, payment schedule, reference rate, etc.)
  for each subset M_k in the partition P **do**
    Scale notionals by dividing by the maximum notional in the subset
    Create a QUBO problem based on Eq. 1
    Solve the QUBO problem using a quantum annealer, finding a group of swaps to be netted G_k, where $G_k \subseteq M_k$
    Create a netting proposal for the group G_k
  end for
  Aggregate all netting proposals
  return Netting proposal for all swaps to be netted

The input to the prototype follows the general layout of the ClearingConfirmed message in the Financial products Markup Language (FpML). It has been tested with two-legged plain vanilla interest rate swaps, and can be expanded to include a wide variety of instruments in the rates category of Over the Counter (OTC) derivatives.

4 Conclusions and Future Work

We have demonstrated that swap netting problems can be solved using a quantum annealer. Our prototype allows for the netting of swaps that differ in their notional and fixed interest rate, but the other economic terms must be the same for all swaps to be netted. However, the incompatibility function we have defined is general, and allows the identification of nettable groups that have variations in any of their economic terms.

Our solution solves the swap netting problem by separation. More specifically, we partition the swaps and calculate the netting proposals independently for each of the subsets. However, the rapidly increasing size of quantum annealers will ultimately allow a simultaneous optimization over the partitions as well. Investigating how to create netting proposals for other combinations of swaps would be an interesting avenue for future research.

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